where the constants κ and λ are chosen to generate the desired transient response. Using these controls the rate of change of the potential of the system is given by

$$\frac{d\tilde{V}}{dt} = -\sum_{i=1}^{N} \kappa_{1i} \dot{l}_i^2 - \sum_{i=1}^{N} \kappa_{2i} \dot{l}_i^2 \tag{16}$$

which is clearly negative definite as required.

Implementation

The method will now be implemented in a simple numerical example. A single ring of 10 satellites will be considered at an operating altitude of 400 km. The nominal spacing for the satellites will then be 36 deg. Each satellite will be perturbed by a random amount, up to 10 km in the azimuthal direction. It will be assumed for ease of illustration that there are no radial positioning errors in the system. Such errors can be easily accommodated through minor modifications to the method.

The controls defined by Eqs. (15) generate small accelerations in the radial and transverse directions that generate an azimuthal drift in each of the 10 satellites. As each satellite moves, its motion perturbs the other nine satellites, leading to a complex nonlinear interaction. However, due to the damping terms in the controls, the potential is monotonically decreasing through these interactions. It should be noted that the potential function of any single satellite is not guaranteed to decrease. Only the total potential of the system will monotonically decrease. Therefore, the partitioning of potential between satellites will vary owing to the nonlinear intersatellite interactions.

The mean intersatellite spacing is defined by the following function:

$$\Delta\bar{\phi} = \frac{1}{N} \sum_{i=1}^{N-1} \phi(i+1) - \phi(i) \tag{17}$$

It can be seen from Fig. 2 that a mean spacing of 36 deg is indeed achieved through the use of these controls. This corresponds to the "minimum energy" configuration of the system. This configuration is then stable against perturbations to the ring. For example, if one satellite in the ring were to fail, the ring would then autonomously reform to generate uniform spacing with the remaining $N-1$ satellites and hence uniform coverage. Similarly, if an on-orbit spare within the ring is activated to replace a failed satellite, the ring will again autonomously reform. Such an autonomous capability may greatly reduce ground segment work loads.

Conclusions

A method has been investigated that allows the autonomous formation of a ring of satellites. The method uses information on the intersatellite spacing to generate low-thrust radial and transverse control accelerations. Using the concept of potential functions, the uniform ring is seen as a minimum energy configuration of the system. The control accelerations ensure that the potential function of the entire system monotonically decreases so that this minimum energy configuration is achieved from any initial configuration. It is believed that such autonomous methods may provide significant operational advantages for future multisatellite rings for global point-to-point communications.

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Orbital Strategies Around a Comet by Means of a Genetic Algorithm

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I. Introduction

THE mission ROSETTA is planned to send a spacecraft to a comet, to rendezvous and explore its features and to land on the surface and take samples. This Note refers to a problem encountered in a previous study[1,2] of computing a complicated sequence of maneuvers to fly over five candidate landing sites while subject to several constraints. Despite the fact that it is initially acceptable to employ classical orbital mechanics approximating the gravitational field as spherical, the dimensionality of the problem and the numerous constraints frustrated an efficient computer algorithm to handle five sites. The use of a genetic algorithm appears, however, to provide a very satisfactory solution.

II. Close Observation of a Comet

A. Specification of the Required Strategy and Constraints

A strategy of maneuvers is required to pass over up to five candidate landing sites at an altitude of 5 km and accomplish the sequence within 20 days subsequent to the following constraints: 1) communication to Earth must not be interrupted by an occultation, 2) orbits should be such that impact does not occur if a maneuver fails, 3) the normal to an orbit plane must not be too close to the direction from spacecraft to Earth in order to preserve a Doppler radiometric ground-based measurement, 4) the landing zones must be illuminated at flyover, and 5) viewing should occur from within 30 deg of the surface normal.

B. Orbital Mechanics

The motion of the comet nucleus is to be regarded as spinning and nutating, although in this Note only spin has been assumed. The extension of the computer program to include nutation would be only a minor modification, viz., preintegration and storage of the Euler equations of rotational motion for a rigid body. As in the earlier study, an irregularly shaped nucleus has been approximated as an ellipsoid, but this could be generalized by means of a representation in terms of a series of spherical harmonics.

As a result of the rotation of the comet nucleus, the specification of a given flyover time over an identified site implies a position vector in nonrotating (ecliptic) axes. Thus, if five flyover times are specified for a given sequence of sites, the trajectory around the comet must pass through five known points. This condition can be satisfied by five maneuvers, and the restriction has been accepted (not necessary) that there is one maneuver in each interval before a flyover time. It follows that for each flyover only the following parameters are necessary: 1) site to be visited and hence a position vector in rotating body axes of the comet nucleus, 2) time of maneuver preceding a given flyover, 3) time of flyover, and 4) orbit integer (+1 or −1) to indicate whether an orbit connects two position vectors by traversing over the subtended angle or 360 deg less than angle.

Further explanation is, however, necessary with respect to the calculation of the orbits passing through the flyover points and starting from given initial conditions. Apart from the use of several standard formulas of conic orbits, a special treatment is needed of Lambert's theorem (Ref. 3, Sec. 7.4), which states that if an orbital transfer occurs from position vector $\boldsymbol{r}_1$ to $\boldsymbol{r}_2$, then the time of transfer t depends

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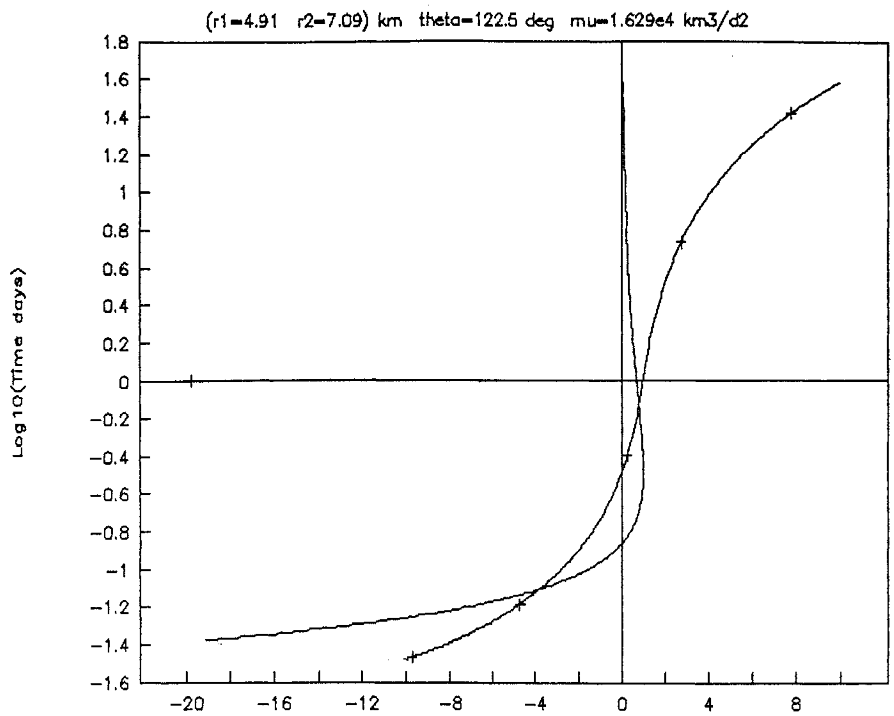

Fig. 1 Transfer time as function of $1/a$ and u; —— $1/a$ or ++++ u [Eqs. (2) and (3)].

only on the semimajor axis a, the chord c joining the two points, and $(r_1 + r_2)$, i.e.,

$$t = F(a, r_1 + r_2, c) \tag{1}$$

This equation must be used here in reverse, i.e., given t, etc., determine a. Even ignoring the singularity of the parabola, the calculation is complicated because there are six forms to the function F for ellipses and hyperbolas and depending on the position of the foci. The transfer time is shown in Fig. 1 (for an example) as a function of $1/a$.

To use an iterative numerical method automatically and reliably, the preceding relationships must be transformed to obtain a monotonic function with a one-to-one relation between t and a. Use an input variable u instead of a such that

$$a = 3(r_1 + r_2 + c)u/8; \qquad u > 1 \tag{2}$$

$$a = (1/4)(r_1 + r_2 + c)/(1 - u^2/3); \qquad u \leqq 1 \tag{3}$$

and select from the cases of function F (Ref. 3, Sec. 7.4) according to 1) elliptical transfers including the apoapsis if $u \geqq 0$ and 2) all other cases if $u < 0$.

Figure 1 shows Log10 (time) plotted also as a function of u, and this was employed to determine a numerically (by the secant method) given t. It is a very robust calculation, as is necessary because the genetic algorithm calls this routine for a large range of parameters, many of which provide extreme solutions of no interest but which must not lead to abortion of computations.

III. Use of a Genetic Algorithm

A. Scale of the Problem

If an objective function is defined as the sum of the magnitudes of all velocity changes (with some allowance to favor viewing time over each site), then this is a nonlinear optimization problem, i.e., five flyover times and five maneuver times have to be adjusted to minimize an objective function subject to the constraints of Sec. II.A. The objective function has, however, numerous local minima, and the constraints are complicated and nonlinear. It is more realistic to regard the problem as one of finding acceptable solutions. Conventional algorithms such as quadratic programming are useful only perhaps to refine one of many feasible solutions.

A workable algorithm was first developed (Ref. 2) by subdividing a time horizon into N subintervals and carrying out a four-dimensional search in terms of two maneuvers and two flyover times. The order of visiting the two sites was fixed but search was also necessary for the orbit integers (+/−1) mentioned earlier. This was acceptable in terms of computer time for two flyovers, but simple calculations reveal that this cannot realistically be extended to four or five flyovers. For example, for 100 subintervals, the computing time increases as 100 raised to the power ($2x$ number of flyovers).

B. Genetic Algorithm

A genetic algorithm is attractive in this case because 1) the computing time does not increase rapidly with the number of flyovers and 2) the numerous constraints are an advantage and not a difficulty. Such algorithms have been applied in recent years to a wide diversity of problems. Readers may, for example, refer to a recent tutorial paper by Grefenstette,[4] an introduction by Stender,[5] or a book edited by Davis.[6] The original and most common formulation is to parameterize the independent variables as a bit string (chromosome), each cell being set to 0 or 1. This was the approach adopted here, and a length of 10 bits per flyover was defined as follows: 1 bit—orbit integer (item 4 in Sec. II.A), 4 bits—interval from previous flyover (or start) to current maneuver, 4 bits—interval from maneuver to flyover, and 1 bit—used with corresponding single bits from the other flyovers to encode a particular sequence of visits to the sites.

In the case of five flyovers, the last bit provides a total of five bits to specify a visiting order. In that case 32 different visiting sequences can be defined; they were actually derived by sampling at random from the 120 (factorial 5) permutations. The total length of the bit string for 5 sites was therefore 50, or 45 if the option of a user-defined fixed visiting sequence was adopted. If desired, two more bits (bringing the total length to 52) could be added to accommodate all 120 permutations of the visiting sequence.

The suite of computer programs was developed in C++ in three parts: 1) initialization of numerous parameters in a "constructor" of a C++ class, including computation and storage (at a sufficiently large number of points) of the rotational motion of the comet nucleus; 2) computation of the objective function for any given bit string involving several standard routines of orbital mechanics, the iterative use of Lambert's theorem (Sec. II.B), and routines to check all of the constraints; and 3) the code of the genetic algorithm.

Use of the class structure of C++ avoided passing numerous parameters to evaluate the objective function, although "modules" of Fortran 90 would be equally convenient. The code of the genetic algorithm was conventional in that it consisted of the following: 1) random initialization of the bit strings of an initial population of 100–300, 2) mating to form the next generation by favoring the fittest as measured by the objective function that included penalty terms when constraints were violated, 3) "crossover" at random,[4,6] and 4) low probability mutation, i.e., random change of a bit from 0 to 1 or vice versa.

Item 3 is an essential feature of genetic algorithms for exploring parameter space while favoring more efficient solutions. Item 4 has only a minor effect but helps to avoid premature convergence at local optima.

IV. Illustrative Results

Extensive parametric studies must be undertaken if this tool is to be utilized for the mission analysis, but only one or two illustrative results can be included in this short paper.

First, it should be clarified that penalty terms were added to the objective function if constraints were violated, but such additions were greater if the violations occurred in the first one or two flyovers. In this way bit strings develop in which at least orbits over the earlier sites satisfy the constraints. Parameters that must be set by trial and error include 1) the population, 2) the crossover probability, and 3) the mutation rate, although other experience of setting such parameters has been well reported. The values of such parameters adopted (after a number of trials) as a baseline were as follows: population = 200, crossover probability = 0.9, and mutation rate = 1 in 200.

The computing time per generation was approximately proportional to the population but increased only moderately as the number of flyovers, e.g., on a 386 machine running at 40 MHz with a coprocessor, 30 and 45 s per generation for 2 and 5 flyovers, respectively.

Bit substrings were converted to times using a subinterval of 1 h, which results in the shortest interval between maneuvers and flyovers being 1 h, and the forward horizon was approximately 7 days for 5 flyovers. Parameters of only one comet have been employed to demonstrate the use of a genetic algorithm; it was the baseline of earlier studies,[1,2] viz., an ellipsoid with semiaxes (3.5, 2.6, 1.7) km spinning about the third axis with a period of 10 h. Principal gravitational factor = 16,291 km^3/day^2.

Some typical results for five flyovers are shown in Fig. 2 for three different initial randomizations of the starting population of bit strings. As in all other tests, simulation up to 20 generations was sufficient because any further progress was negligible. However, although the convergence on the logarithmic scale is very satisfac-

Table 1 Details of final results of the three different randomized simulations of Fig. 2

	Site					Total
	1	2	5	3	4	m/s
Maneuver days[a]	0.292	1.167	2.000	2.583	3.625	3.6
Flyover days[b]	0.708	1.833	2.125	3.208	4.167	—
Viewing hours[c]	1.4	1.0	3.4	2.3	16.3	—
Maneuver days[a]	0.292	0.958	1.917	2.750	3.792	4.0
Flyover days[b]	0.708	1.417	2.125	3.292	4.417	—
Viewing hours[c]	1.4	1.0	1.2	2.1	5.3	—
	1	3	5	4	2	
Maneuver days[a]	0.083	1.125	2.208	3.292	4.000	3.1
Flyover days[b]	0.667	1.583	2.833	3.583	4.333	—
Viewing hours[c]	1.2	2.6	2.2	1.5	1.8	—

[a]Maneuver days = time from start at which maneuvers occur.
[b]Flyover days = time from start at which flyovers occur.
[c]Viewing hours = time over site when viewing is within 30 deg of the surface normal.

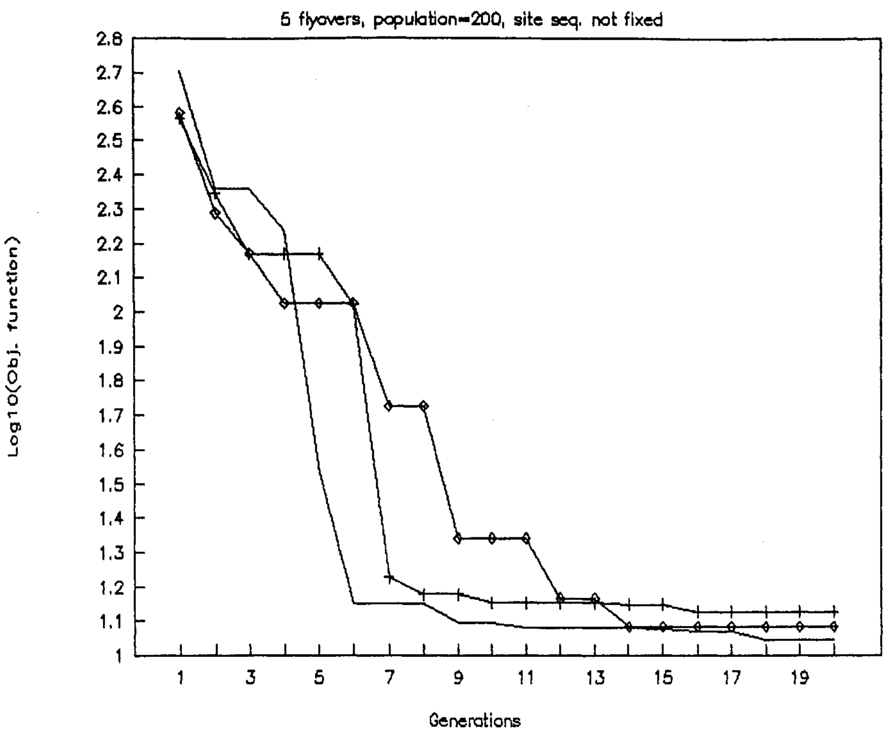

Fig. 2 Convergence of genetic algorithm; — ran = 0, + ran = 1, ◊ ran = 2.

tory, the final solutions are different as summarized in Table 1. This behavior persists with greater populations or higher mutation rates.

V. Conclusions

1) Use of a genetic algorithm permits a very efficient algorithm that, for five flyover sites and a number of awkward constraints, is not feasible by a search method or nonlinear optimization.

2) Even though dissimilar solutions arise for different randomized initial populations, all of the solutions are highly acceptable and surprisingly efficient in terms of the magnitude of velocity changes and time elapsed.

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Clarification of the Garber Instability for Gravity-Gradient Stabilized Spacecraft

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Introduction

GARBER[1] demonstrated the effect of a constant disturbing torque upon the transient response of a gravity-gradient stabilized satellite. In particular, he concluded that a bias angle in pitch, due to a pitch disturbing torque, leads to instability in the roll-yaw motion. As part of the design process of passively damped, gravity-gradient stabilized satellites, analysts evaluate their designs to avoid the so-called "Garber instability" in the presence of aerodynamically induced pitch bias angles.[2–4] For the Long Duration Exposure Facility, avoidance of the Garber instability was the primary consideration in sizing the passive damping devices used, in spite of the fact that their high-fidelity simulations failed to corroborate predictions made by Garber's equations.[4]

Garber's results apply correctly for a satellite under the influence of a constant body-fixed torque (the model used in his derivation) and approximately for an aerodynamically induced torque consistent with a pure specular reflection flow model that produces forces normal to the spacecraft's surface. However, aerodynamic torques in low Earth orbit are not body fixed and are more closely approximated by a pure diffuse reflection flow model that produces a force opposite to the relative velocity vector, i.e., pure drag.[5] A drag-induced torque, arising from a center-of-mass to center-of-pressure offset along the body's nadir pointing principal axis, produces a pitch bias angle but does not lead to an instability. Therefore, the use of Garber's equations when the pitch bias angle is primarily due to aerodynamic drag will produce erroneous results. Furthermore, it is not the bias angle that leads to the instability; it is the torque mechanism. For example, a body-fixed torque produces the same instability for a bias angle of zero, a condition that could exist if a drag torque nominally cancels the body-fixed torque.

This Note derives the linearized equations of motion for a gravity-gradient stabilized spacecraft in the presence of both body-fixed and drag-induced disturbance torques and discusses some of the results that follow from them.

Linearized Equations of Motion

The simplified aerodynamic torque model used here assumes that all aerodynamic torques are due to drag; i.e., lift is negligible. The drag force acts at the center-of-pressure of a flat plate with the normal to the plate along the body x axis. The center-of-pressure is assumed fixed in body axes. A change in the orientation of the spacecraft causes a change in the drag-induced torque in two ways: movement of the center-of-pressure (c.p.) relative to the local-vertical local-horizontal (LVLH) frame and a change in the magnitude of the drag force due to a change in the exposed area of the plate. The LVLH reference frame, denoted n, is defined as x along the velocity vector, z down, and y forming an orthogonal right-handed set.

The moment-of-inertia tensor in body axes is $\boldsymbol{J}^{\text{body}}$. Another coordinate frame b is defined that is fixed to the body and coincident with the LVLH reference frame when the spacecraft is at its torque equilibrium attitude (TEA). As will be seen, the introduction of the b frame permits simple expressions for the linear equations even for arbitrarily large deviations between the body and LVLH frames. The moment-of-inertia tensor in this frame is $\boldsymbol{J}^b = \boldsymbol{C}^b_{\text{body}}\boldsymbol{J}^{\text{body}}\boldsymbol{C}^{\text{body}}_b$, where $\boldsymbol{C}^b_{\text{body}}$ is the constant direction cosine matrix transforming from the body frame to the b frame. In the derivation that follows $\boldsymbol{C}^k_j$ is a direction cosine matrix transforming from frame j to frame k, ω^m_{jk} is the angular rate of frame k relative to frame j, components taken in frame m, and Ω^m_{jk} is the skew symmetric matrix form of ω^m_{jk}. Superscripts denote the frame components of vectors or tensors are taken in. The following equations also assume a circular orbit, small attitude motions about the TEA, angular rates relative to the LVLH frame that are small compared with orbital angular rate, and a constant moment-of-inertia tensor in any body-fixed frame.

The angular momentum $\boldsymbol{H}^i$ of the spacecraft expressed in the inertial frame is

$$\boldsymbol{H}^i = \boldsymbol{C}^i_n\boldsymbol{J}^n\left(\omega^n_{in} + \omega^n_{nb}\right) \tag{1}$$

Differentiating and equating to the sum of all external torques

$$\Sigma\boldsymbol{T}^i = \dot{\boldsymbol{H}}^i = \dot{\boldsymbol{C}}^i_n\boldsymbol{J}^n\left(\omega^n_{in} + \omega^n_{nb}\right) + \boldsymbol{C}^i_n\dot{\boldsymbol{J}}^n\left(\omega^n_{in} + \omega^n_{nb}\right) + \boldsymbol{C}^i_n\boldsymbol{J}^n\left(\dot\omega^n_{in} + \dot\omega^n_{nb}\right) \tag{2}$$

Substituting $\dot{\boldsymbol{C}}^i_n = \boldsymbol{C}^i_n\Omega^n_{in}$ and $\dot{\boldsymbol{J}}^n = \Omega^n_{nb}\boldsymbol{J}^n - \boldsymbol{J}^n\Omega^n_{nb}$ and noting that $\omega^n_{in} = [0\ \ -\omega_0\ \ 0]^T = \text{const}$, where ω_0 is the magnitude of orbital angular rate, gives

$$\dot\omega^n_{nb} = [\boldsymbol{J}^n]^{-1}\left[\Sigma\boldsymbol{T}^n - \left(\Omega^n_{in}\boldsymbol{J}^n + \Omega^n_{nb}\boldsymbol{J}^n - \boldsymbol{J}^n\Omega^n_{nb}\right)\left(\omega^n_{in} + \omega^n_{nb}\right)\right] \tag{3}$$

The external torques consist of the gravity-gradient torque $\boldsymbol{T}^n_{gg}$, the drag torque $\boldsymbol{T}^n_d$, and a body-fixed torque $\boldsymbol{T}^n_{bf}$. These are given by

$$\boldsymbol{T}^n_{gg} = 3\omega_0^2\begin{bmatrix}0\\0\\1\end{bmatrix} \times \boldsymbol{J}^n\begin{bmatrix}0\\0\\1\end{bmatrix} \tag{4}$$

$$\boldsymbol{T}^n_d = \boldsymbol{r}^n \times \boldsymbol{D}^n = \boldsymbol{C}^n_b\boldsymbol{r}^b \times \boldsymbol{D}^n \tag{5}$$

$$\boldsymbol{T}^n_{bf} = \boldsymbol{C}^n_b\boldsymbol{C}^b_{\text{body}}\boldsymbol{T}^{\text{body}}_{bf} \tag{6}$$

where $\boldsymbol{r}^b = [r_x\ \ r_y\ \ r_z]^T$ is the vector distance from the center-of-mass to the center-of-pressure and is assumed constant in the b frame and $\boldsymbol{D}^n$ is the drag force vector.

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